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1. **Explain the conditions for the existence of the Fourier transform.**

**Ans:**

A function f(t)'s Fourier Transform may be found using:

F(ω)=dt

In order for this integral to exist, f(t) has to meet several requirements:

**(I) Absolute Integrability (Dirichlet’s Condition)**

A completely integrable function f(t) is required:

By doing this, the Fourier integral is guaranteed to converge rather than diverge to infinity.

**(ii) Piecewise Continuity**

In every finite interval, the function f(t) ought to be piecewise continuous. This implies:

* The function should be continuous except at a finite number of points.
* Any discontinuities must be finite (not infinite jumps).

**(iii) Bounded Variation**

Any finite interval's total variation of f(t) ought to be limited. This helps in avoiding functions with excessive oscillations.

The Fourier transform exists and offers a useful frequency domain representation of f(t) if certain prerequisites are satisfied.

1. **Why Does DFT Produce Both Positive and Negative Frequencies?**

**Ans:**

A discrete signal x[n] has the following Discrete Fourier Transform (DFT):

X[k]=

where the frequency index is denoted by k.

**(i) Sinusoidal Representation in DFT**

There are both positive and negative exponentials in a sinusoidal function:

The DFT naturally depicts both positive and negative frequencies since it is calculated using complex exponentials.

**(ii) Periodicity of the DFT**

The DFT has a period of N. There is symmetry in the frequency spectrum:

X[k]=X[N-k]

This indicates that the spectrum's negative frequency components are a result of the transform's mathematical structure.

**(iii) Interpretation**

The mathematical framework of Fourier analysis is the reason why negative frequencies appear in most real-world signals, even though they have no physical significance. Real signals have a symmetric spectrum because the positive and negative frequency components are complex conjugates of one another.

1. **What is the sampling theorem? Derive the Nyquist rate.**

**Ans:**

According to the Sampling Theorem:

"A band-limited continuous signal can be completely represented by its discrete samples if it is sampled at a rate greater than or equal to twice its highest frequency component."

**(i) Mathematical Representation**

Let x(t) be a continuous-time signal with a maximum frequency fmax

The sampled version is:

x[n]=x(nTs)

where Ts = is the sampling period.

To avoid loss of information (aliasing), the sampling frequency fs must satisfy:

Fs2fmax

This minimum frequency fs = 2fmax is called the Nyquist rate.

**(ii) Derivation of the Nyquist Rate**

* The Fourier Transform of x(t) is X(f), with frequency components up to fmax.
* Sampling x(t) creates a periodic spectrum in the frequency domain.
* If fs<2fmax, overlapping (aliasing) occurs, making it impossible to reconstruct the original signal.
* The critical boundary occurs when fs = 2fmax, ensuring no aliasing.

Thus, the Nyquist rate 2fmax is the minimum required sampling frequency.

1. **Define stability in terms of system response.**

**Ans:**

A system is stable if its output remains bounded for any bounded input. This is known as Bounded Input, Bounded Output (BIBO) Stability.

**(i) BIBO Stability Condition**

A system with impulse response h(t) is stable if:

For a discrete-time system with impulse response h[n], stability requires:

**(ii) Intuition Behind Stability**

* If the impulse response is too large or grows indefinitely, the system response can become unbounded.
* If the system is linear and time-invariant (LTI), its stability depends entirely on the behavior of h(t).
* If the poles of the system’s transfer function lie inside the unit circle (discrete systems) or in the left-half plane (continuous systems), the system is stable.

**5. Properties of Fourier Transform**

The Fourier Transform has several important properties:

**(i) Linearity**

Scaling and addition in the time domain correspond to the same operations in the frequency domain.

**(ii) Time Shifting**

A shift in the time domain introduces a phase shift in the frequency domain.

**(iii) Frequency Shifting**

Multiplying by an exponential shifts the spectrum.

**(iv) Time Revising**

Scaling in time compresses or expands the frequency spectrum.

**(v) Modulation Theorem**

Convolution in the time domain corresponds to multiplication in the frequency domain.

**(vi) Convolution Theorem**